## Monopoly and elasticity

Econ 201/Haworth

We've established that, unlike perfectly competitive firms, a monopoly firm faces a downward sloping demand curve. Of course, as the only seller, this would also be the market demand curve. In other words, the monopoly firm serves the entire market, so the firm's demand curve and market demand curve would be the same curve.

Here's what that demand curve (and associated marginal revenue curve) might look like:
$\mathrm{P}=100-2 \mathrm{Q} \quad$ (where $\mathrm{P}=$ market price, $\mathrm{Q}=$ market quantity)
$M R=100-4 \mathrm{Q}$

Note that we can use upper case Q, which has always represented the market output, or lower case q, which has always represented the firm's output, since this firm is the only supplier.

Here are the monopolist's demand and MR curves on a graph.


Note that the value of the vertical intercept from the equations above (pt A) is also included on the graph. This point is sometimes called the "reservation price" and tells us how high this price can go before demanders stop buying the good being sold.

Let's recall what we've said about elasticity, specifically own price elasticity. The own-price elasticity (which we'll just call "the elasticity" from here on out) is measured as the percentage change in quantity sold $(\mathrm{Q})$ divided by the percentage change in price. That measure is provided below as equation (1).
(1) $\varepsilon_{D}=\frac{\% \Delta Q}{\% \Delta P}$

In the elasticity section, we broke down an equation like (1) by substituting in an equation for how to calculate these two percentage change values. Here they are below as (2) and (3).
(2) $\% \Delta Q=\frac{\Delta Q}{Q}$
(3) $\% \Delta P=\frac{\Delta P}{P}$

If we substitute (2) and (3) into (1), then here's the rather complicated-looking result:

$$
\begin{equation*}
\varepsilon_{D}=\frac{\left(\frac{\Delta Q}{Q}\right)}{\left(\frac{\Delta P}{P}\right)} \tag{4}
\end{equation*}
$$

Note that this is just a fraction divided by a fraction, so we can simplify it and get:
(5) $\varepsilon_{D}=\left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{P}{Q}\right)$

Notice that first term in the parenthesis above, $\frac{\Delta Q}{\Delta P}$. That term could also be described as any movement along the demand curve that changes Q divided by the associated movement along that demand curve which changes $P$. In other words, it's run/rise, which is just the inverse of slope. Therefore, if we call the slope of the demand curve " $m$ ", then (5) can be rewritten as:
(5') $\quad \varepsilon_{D}=\left(\frac{1}{m}\right)\left(\frac{P}{Q}\right)$

This equation can take on greater meaning when we consider the demand curve equation from above (i.e. $\mathrm{P}=100-2 \mathrm{Q}$ ). That equation reveals the slope of our current demand curve, and also provides us with a means of calculating Ps and/or Qs. In other words, if we plug the slope from this demand curve into ( $5^{\prime}$ ), then we can start plugging in values for P or Q to get the coordinates for a bunch of different points on the demand curve. We can use those coordinates along with the slope and (5') to calculate the elasticity of every point on this demand curve.

Note that in the graph on the previous page there are a series of points labeled on the graph. Pt A was already mentioned as the vertical intercept, but you have additional points as well (pts B, C, etc.). Assume that we know the price at each of those points. If so, then we can plug that price into our demand curve equation and get a quantity. I.e., we can plug Ps into our equation for demand and get the Qs which are associated with each P.

We'll assume a series of prices (provided in the table below) and we'll use the demand curve equation to calculate our quantities. That information is included in columns 2 and 3 in the table below..

| Point | Price | Quantity | Elasticity |
| :---: | :---: | :---: | :---: |
| B | 90 | 5 | $\left\|\varepsilon_{\mathrm{D}}\right\|=9.0$ |
| C | 70 | 15 | $\left\|\varepsilon_{\mathrm{D}}\right\|=2.3$ |
| D | 50 | 25 | $\left\|\varepsilon_{\mathrm{D}}\right\|=1.0$ |
| E | 40 | 30 | $\left\|\varepsilon_{\mathrm{D}}\right\|=0.67$ |
| F | 30 | 35 | $\left\|\varepsilon_{\mathrm{D}}\right\|=0.43$ |
| G | 20 | 40 | $\left\|\varepsilon_{\mathrm{D}}\right\|=0.25$ |

Remember that the slope of this demand curve is -2 (i.e. $\mathrm{m}=-2$ ). If we plug -2 in for m in equation ( $5^{\prime}$ ) above, and for each row of the table, we plug that particular P and Q into ( $5^{\prime}$ ), then the result is a value for the elasticity associated with that particular point. Those values are provided in the final column of the table (note that we presented them in terms of absolute value, because own-price elasticity is always negative). Plugging those results into our graph from above, we have this.


What do we observe on this graph? Note how the values for elasticity change along the graph. They start at pt G with 0.25 and begin increasing as you move up the demand curve. This suggests that the points on a demand curve become more and more elastic as you move up the curve. Note as well that they are inelastic (less than 1) below pt D and elastic (greater than 1) above pt D . Pt D itself is unit elastic (equal to 1 ). If we realize that the firm would never produce an output level greater than $\mathrm{Q}_{\mathrm{MAX}}($ where $\mathrm{MC}=0$ ), then we know that the firm will never set a price lower than $\mathrm{P}_{\text {MIN. }}$. since $\mathrm{P}_{\text {MIN }}$ is the price associated with producing $\mathrm{Q}_{\text {max }}$ ). Of course, that assumes the monopoly is using $\mathrm{MR}=\mathrm{MC}$ to set $\mathrm{Q}^{*}$ and $\mathrm{P}^{*}$.

